Accurate Estimation of Aircraft Inertia Characteristics from a Single Suspension Experiment

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A novel method for the experimental determination of the aircraft center of gravity and the moments and products of inertia is presented and evaluated. The method is based on the application of statistical parameter estimation techniques to the analysis of multi-degree-of-freedom oscillations using a high-accuracy instrumentation system. An existing suspension rig, in which the aircraft can simultaneously rotate about the *Y* and *Z* axes (pitch and yaw axes) and translate along the *X* (roll) and *Y* axes, is used to demonstrate the validity of the method. The inertia moments *Iy* and *Iz*, the inertia product *Iyz*, and the three coordinates of the center of gravity are simultaneously estimated from measurements of only one oscillation. The experiment is much less time-consuming compared to earlier methods, since no separate center-of-gravity measurements nor rig reconfigurations are needed. It is suggested that the remaining inertia characteristics *(Ix, Ixz,* and *Ixy)* can be simultaneously estimated after a slight modification in the suspension construction.

Nomenclature

$A_{x_M}, A_{y_M}, A_{z_M}$	= components of specific forces along the *X, Y,* and *Z* axes of F_M; see Fig. 2
c.g.	= aircraft center of gravity (including experimental components)
cga	= aircraft center of gravity (excluding experimental components)
d	= half the distance between the pendulum arms; see Fig. 3
d_X, d_Y, d_M, d_N	= aerodynamic damping coefficients
e	= distance from the c.g. to the geometrical plane of symmetry; see Fig. 3
F_I, F_B, F_M, F_A	= inertial (*I*), horizontal body-fixed (*B*), measurement (*M*), and aircraft datum (*A*) reference frame
g	= acceleration due to gravity
Iy_B, Iz_B, Iyz_B	= moments and product of inertia of aircraft, including experimental components, with respect to the axes of F_B
$Ix_A, Iy_A, Iz_A, Iyz_A, Ixz_A, Ixy_A$	= moments and products of inertia with respect to the body axes (parallel to F_A) through cga
m	= aircraft mass (including experimental components)
p_M, q_M, r_M	= components of body-rotation rates about the *X, Y,* and *Z* axis of F_M; see Fig. 2
R	= generalized coordinate; see Fig. 3
R_o, R_l	= pendulum length for longitudinal and lateral motion
r	= distance between the c.g. and the line through the lower knife edges; see Fig. 3
v_o	= vector of measurement bias errors
x_1	= initial state vector
ζ_{cga}, ζ_M	= position vectors relative to F_A of cga and origin of F_M
σ, τ	= generalized coordinates; see Fig. 3
ψ, θ, φ	= Euler angles for axes of F_B (generalized coordinates); see Fig. 3. Index *M* denoting Euler angles for F_M in equilibrium position
θ_A	= nominal aircraft pitch angle

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Introduction

ACCURATE estimates of aircraft inertia characteristics—the location of the center of gravity, the moments and products of inertia—are crucial in linear and nonlinear simulation, analysis of stall-spin characteristics, control system design, and dynamic flight testing. Throughout the past 60 years, experimental techniques for the determination of inertia characteristics have been developed and continuously improved.[1-7] These techniques have the following in common: the rolling, pitching, and yawing moments of inertia are derived from the measured characteristic frequencies of one-degree-of-freedom oscillations about the corresponding axes. The resulting data have to be corrected for the distance between the axis of oscillation and the corresponding body axis through the aircraft center of gravity. Separate experiments must be carried out to locate the center of gravity. For the determination of the inertia product in the *XZ* plane of symmetry, additional oscillation measurements are required at different aircraft attitudes. In all cases, a symmetrical mass distribution is assumed, and the effects of oscillatory damping is neglected.

Although the literature has been concerned with improved techniques with respect to accuracy, no significant reduction has been gained regarding the cumbrousness of these methods. The separate one-degree-of-freedom oscillations and center of gravity measurements require time-consuming reconfigurations of the experiment setups. Extreme care in conducting the various experiments is considered essential to obtain good results.

The method proposed in this paper eliminates the need for different experiment setups and is much simpler in operation. The aircraft is allowed to execute general oscillations with more than one degree of freedom. Direct measurement of characteristic frequencies is not attempted, but statistical parameter estimation techniques are applied to estimate the inertia characteristics from the oscillation measurements. Whereas the restriction to one-degree-of-freedom oscillations caused handling problems in previous methods (e.g., Ref. 6), excitation of more than one mode of oscillation is required in the new method. It is seen that, provided the suspension system is properly designed and the modes are adequately excited, all inertia characteristics (including the characteristics representing an asymmetrical mass distribution) can be simultaneously estimated from the oscillation measurements using only one suspension configuration. Effects of aerodynamic damping can be explicitly accounted for.

This paper is organized as follows. First, the suspension rig and the instrumentation system are described. Next, a mathematical model is derived for small aircraft oscillations. The parameters in the model equations represent the inertia characteristics. Subsequently, the parameter estimation problem is precisely formulated. A computer simulation yields a first insight into the excitation requirements of the system and the identifiability of the parameters.

Finally, the results of actual oscillation experiments are presented and discussed. Small model errors are identified and, where possible, corrected for. The parameters of the refined model are accurately reproduced. However, not all inertia characteristics can yet be derived from these estimates. This is due to constraints on roll motions imposed by the suspension construction used and is not a restriction of the method in general. Suggestions are given for construction modifications. The last section is a discussion of the most important results obtained.

Suspension Rig and Instrumentation System

Suspension Rig

The idea for the new method originates from experiments with the rig construction of the Aerospace Department at the Delft University of Technology.[7] The suspension as shown in Fig. 1, where the rig is in the configuration of a bifilar pendulum, permits the aircraft to rotate about the pitch and yaw axes, as well as to translate along the roll and pitch axes, thus constituting a system with four degrees of freedom. The resulting four modes of oscillation are identified as the pitching, yawing, longitudinal-sway, and lateral-sway modes.

Knife-edge bearings and flexible pendulum arms are applied to reduce mechanical friction to the point of insignificance. This certainly holds for the longitudinal translation and the pitch and yaw rotations. The construction is not specifically designed to allow a lateral translation. The pin hinges for this translation (see details in Fig. 1) are subject to significant Coulomb friction. This results in a nonsmooth rotation for large lateral amplitudes. For small amplitudes, the pin hinges do not rotate, in which case the lateral motion is possible only when the pendulum arms are bent just below the upper link and above the lower link.

The bifilar-pendulum construction does not permit aircraft rotations about the roll axis. Thus, oscillation measurements yield no information about the rolling moment of inertia. If measurements are made of oscillations with different nominal aircraft pitch angles, however, it should be possible to derive the rolling moment of inertia Ix (as well as the products of inertia Ixz and Ixy) from the resulting set of data. To allow different nominal pitch angles, the suspension beam (the beam on which the lower knife edges are fixed; see Fig. 1) is made transferable along the longitudinal axis.

Instrumentation System

The flight-test instrumentation system described in Ref. 8 is used for the accurate measurement of the body-rotation rates p_M, q_M, r_M and the specific forces $A_{x_M}, A_{y_M}, A_{z_M}$ (see Fig. 2). The measurement outputs are sampled by a scanner at a rate of 20 samples/s for each channel. Linear fourth-order antialiasing filters are applied with a linear phase characteristic in the frequency range of interest, resulting in a constant and equal time delay (0.05 s) for all signals. The resolution of the instrumentation system amounts to 0.001 deg/s for the rotation rates, and 0.002, 0.001, and 0.003 m/s^2 for the components of the specific forces along the $X, Y,$ and Z axes, respectively.

Mathematical Model for Small Oscillations

Assumptions and Definitions

The geometry of the aircraft-bifilar-pendulum system is shown in Fig. 3. XYZ_I denotes a horizontal rectangular inertial reference frame F_I. The origin of F_I is situated between the suspension points P and Q. The XZ_I plane coincides with the aircraft geometrical plane of symmetry when the system is at rest (equilibrium position). The pendulum arms P–P' and Q–Q' are considered to be of constant length R_o. The mass of the pendulums is assumed to be negligible compared to the mass of the aircraft. The suspension points P, Q, P', and Q' are regarded as (frictionless) ball hinges. Longitudinal and lateral motions of the system are represented by the angles σ and τ, respectively. The "forcing-up effect," resulting from yaw perturbations from the equilibrium position, is represented by the (nonpositive) variable R.

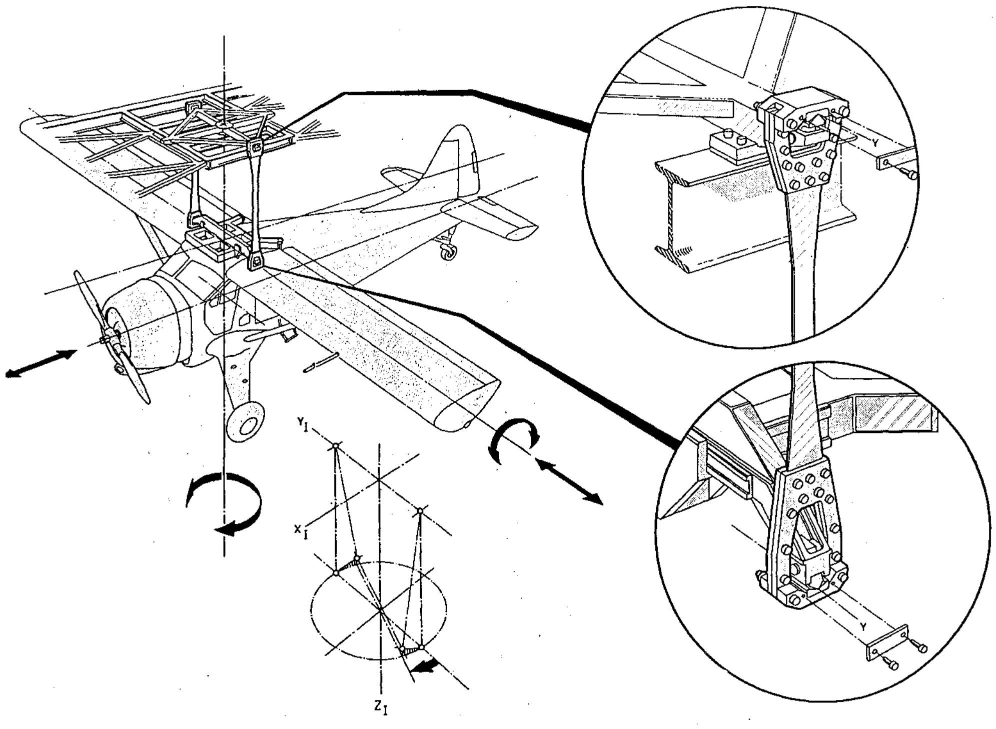

Fig. 1 The bifilar pendulum suspension system; four degrees of freedom.

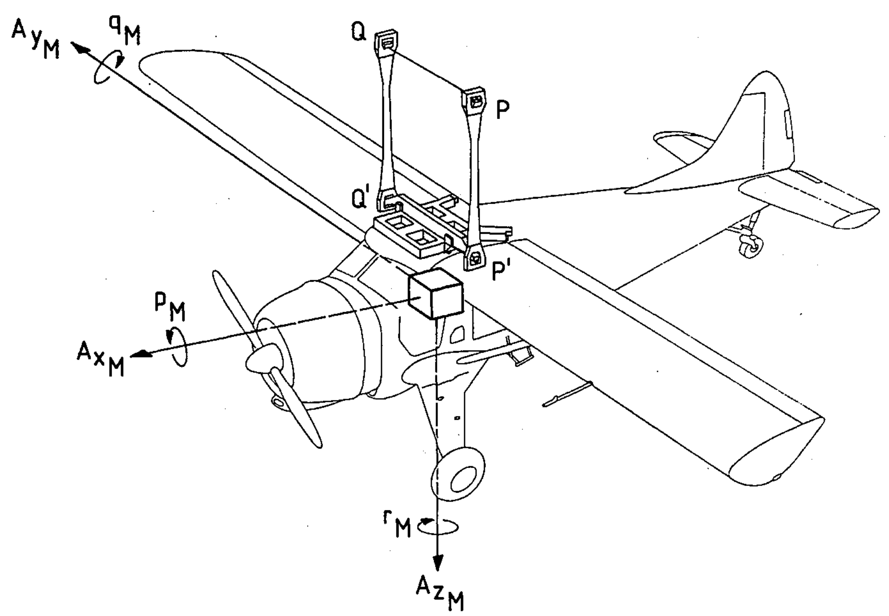

Fig. 2 Measurements of the aircraft motions; specific forces A_{xM}, A_{yM}, and A_{zM}, and rotation rates p_M, q_M, and r_M.

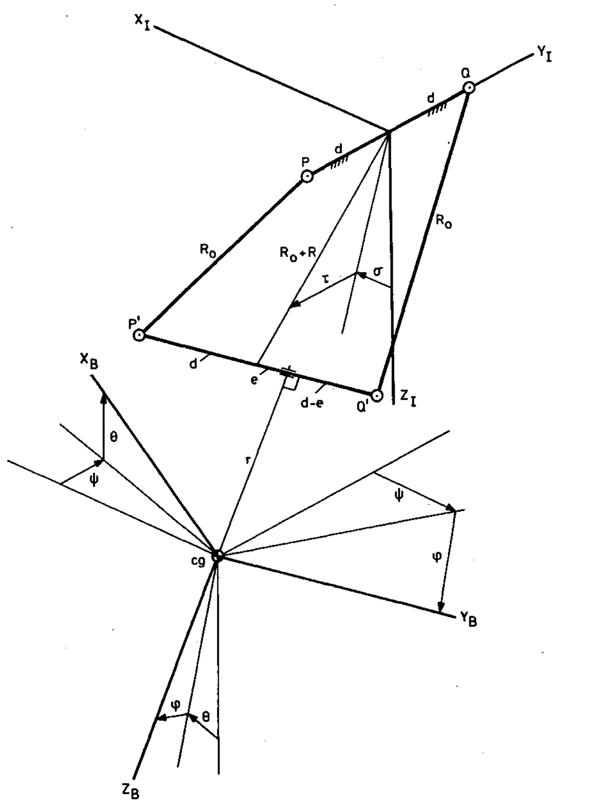

Fig. 3 Geometry of the aircraft-bifilar-pendulum system; inertial reference frame XYZ_I and aircraft body-fixed reference frame XYZ_B.

The aircraft is regarded as a rigid body. XYZ_B denotes a rectangular aircraft body-fixed reference frame F_B. The origin of F_B coincides with the center of gravity (c.g.). The orientation of F_B relative to F_I is defined by the Euler angles ψ, θ, and φ. The axes of F_I and F_B are parallel when the aircraft is in the equilibrium position. The vertical position of the c.g. is defined by the parameter r. It is not assumed that the mass distribution of the aircraft is symmetrical. This results in the c.g. being located at a certain distance e from the XZ_I plane when the aircraft is in the equilibrium position.

Equations of Motion

The equations of motion are derived by applying Lagrange's formulation of the equations of motion of multiple body systems[9]

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\delta T}{\delta \dot{q}_i}\right)-\frac{\delta T}{\delta q_i}=-\frac{\delta V}{\delta q_i}+\sum_{j=1}^{k}\lambda_j\frac{\delta f_j}{\delta q_i}+Q_i \quad i=1,\ldots,n \qquad (1)$$

where T and V denote kinetic and potential energy, and Q_i represents the generalized external force corresponding to the generalized coordinate q_i. In the present case, Q_i represents the effect of aerodynamic damping forces. The number of degrees of freedom of the unconstrained system is denoted by n ($n=6$ if the aircraft is considered as a rigid body). According to Fig. 3, a possible set of generalized coordinates is

$$[q_1 q_2 q_3 q_4 q_5 q_6] = [R\sigma\tau\psi\theta\varphi]$$

representing perturbations from the equilibrium state. In Eq. (1), λ_j denotes a Lagrange multiplier corresponding to a kinematic constraint equation of the form

$$f_j(q_1,q_2,\ldots,q_n)=0 \qquad (2)$$

By the assumption of constant-length pendulum arms, the lower bearings P' and Q' are restricted to moving on spheres of radius R_o, with centers in P and Q, respectively. This gives rise to two independent constraint equations [$k=2$ in Eq. (1)]. The two kinematic constraints lead to a reduction in the number of degrees of freedom to four.

Equations (1) and (2) form a set of eight equations for the generalized coordinates q_i and the Lagrange multipliers λ_j. From this set, the equations of motion are derived by first expressing kinetic and potential energy T and V and the constraint equations (2) in terms of the generalized coordinates q_i and their derivatives with respect to time, then substituting the results in the Lagrange equations (1), and subsequently linearizing the Lagrange equations for small perturbations from the equilibrium state. Thus, linearized differential equations are obtained in terms of those generalized coordinates that correspond to the remaining degrees of freedom. Details of the derivations can be found in Ref. 10. The resulting set of linear differential equations can be written as

$$\begin{bmatrix} R_o^2 & 0 & -R_o e & R_o r \\ 0 & R_o^2 & 0 & 0 \\ -R_o e & 0 & Iz_B/m+e^2 & -Iyz_B/m-re \\ R_o r & 0 & -Iyz_B/m-re & Iy_B/m+r^2 \end{bmatrix} \begin{bmatrix} \ddot{\sigma} \\ \ddot{\tau} \\ \ddot{\psi} \\ \ddot{\theta} \end{bmatrix}$$

$$= -g \begin{bmatrix} R_o & 0 & -e & 0 \\ 0 & R_o & 0 & 0 \\ -e & 0 & d^2/R_o & 0 \\ 0 & 0 & 0 & r \end{bmatrix} \begin{bmatrix} \sigma \\ \tau \\ \psi \\ \theta \end{bmatrix} + \frac{1}{m} \begin{bmatrix} R_o X \\ -R_o Y \\ N-eX \\ M+rX \end{bmatrix} \tag{3}$$

in which X and Y denote aerodynamic (damping) forces along the X_B and Y_B axes and M and N denote aerodynamic (damping) moments about the Y_B and Z_B axes, respectively. The following model is postulated for the aerodynamic forces and moments:

$$X = d_X V_X^2 \operatorname{sign}(V_X) \qquad Y = d_Y V_Y^2 \operatorname{sign}(V_Y)$$

$$M = d_M \dot{\theta}^2 \operatorname{sign}(\dot{\theta}) \qquad N = d_N \dot{\psi}^2 \operatorname{sign}(\dot{\psi}) \tag{4}$$

where V_X and V_Y denote the components of the velocity of the c.g. along the X_B and Y_B axes. The (dimensional) aerodynamic damping coefficients d_X, d_Y, d_M, and d_N are assumed to be constant for a given air density and nominal orientation of the aircraft. The sign functions are used to account for the alternating directions of the velocity and rotation rate components. Note that only the dominant aerodynamic effects are included in the above damping model. Smaller effects, such as transverse aerodynamic forces on the vertical tail plane resulting from yaw motions and vice versa (yaw moments resulting from transverse motions) are assumed to be negligible since the aerodynamic forces are small compared to components of the force of gravity acting on the system.

The set of equations of motion (3) consists of four second-order differential equations, agreeing with the number of degrees of freedom of the system. The second equation of the set represents the decoupled lateral-sway mode. If the aircraft has a symmetrical mass distribution (i.e., if $e = Iyz_B = 0$), then the yawing mode (the third equation) is also decoupled. In that case, the first and last equations describe the symmetrical motions of an independent double pendulum system, consisting of the longitudinal-sway mode and pitch mode. The latter modes are always mutually coupled.

State-Space Model

After the introduction of a state vector

$$x = [\sigma \tau \psi \theta \dot{\sigma} \dot{\tau} \dot{\psi} \dot{\theta}]^T$$

the equations of motion (3) may be written in standard state-space notation

$$\dot{x} = Fx + Gw \tag{5}$$

in which the vector w represents the effect of aerodynamic damping

$$w = [R_o X - R_o Y N - eX M + rX]^T/m$$

Equation (5) may be interpreted as a linear dynamic system. It must be emphasized that only responses on initial conditions are considered; the vector w is introduced in the state-space model merely to account for the effect of aerodynamic damping. The model formulated in this way does not restrict the application of standard parameter estimation techniques.

The output of the system depends on the number, type, position, and orientation of transducers selected to measure the system response. As mentioned, measurements are made of the body rotation rates p_M, q_M, r_M and the specific forces A_{xM}, A_{yM}, A_{zM} (see Fig. 2). These quantities are measured about and along the axes of the rectangular measurement reference frame F_M. The position of F_M relative to the aircraft datum reference frame F_A is defined by the constant vector ζ_M (see Fig. 4). The orientation of F_M, when the system is at rest, is defined by the three constant Euler angles ψ_M, θ_M, and φ_M. After the introduction of the output vector,

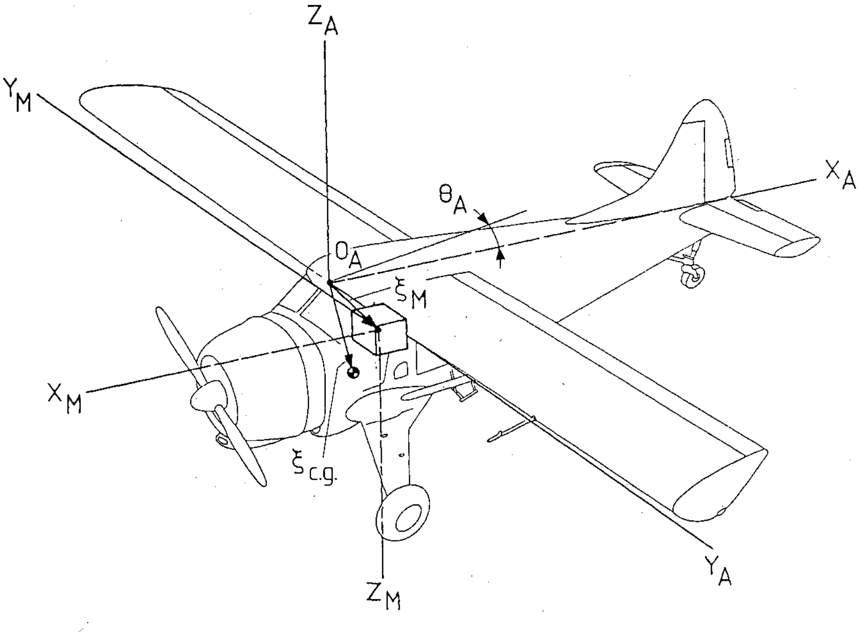

Fig. 4 Definition of the aircraft datum reference frame F_A and the measurement reference frame F_M.

$$y = [p_M q_M r_M A_{x_M} A_{y_M} A_{z_M}]^T$$

the output equation may be written as

$$y = Hx + Kw + y_o \qquad (6)$$

in which the vector y_o represents the (constant) contribution of the force of gravity to the specific force measurements.

Formulation of the Estimation Problem

Maximum Likelihood Estimate

The state-space model equations (5) and (6) contain the unknown system parameters that are to be estimated from a set of discrete-time observations $[y_m(t_i),\ i=1,\ldots,N]$ of the system output $y(t)$ following the response on an initial state $x_1 = x(t_1)$. If only additive disturbances are assumed, the observation model may be written as

$$y_m = y + v_t \qquad (7)$$

in which v_t denotes the vector of measurement errors

$$v_t = v + v_o \qquad (8)$$

The elements of the constant vector v_o represent constant bias errors in the measurements. With the usual assumption that the set of remaining errors $[v(t_i),\ i=1,\ldots,N]$ can be represented by an independent, zero-mean, and Gaussian random process, it is possible to compute the conditional probability density function of the observations[11] for a given value of the parameters. This probability function, evaluated for a given sequence of observations and considered as a function of the parameters, is called the likelihood function of the parameters. The maximum likelihood estimate is the parameter vector that globally maximizes the likelihood function.

Optimization Algorithm

The modified Gauss-Newton method[12] is used to maximize the likelihood function. At the core of this method is the inversion of the so-called information matrix.[13] The diagonal elements of the inverted information matrix are the well-known Cramer Rao lower bounds (the estimator variances) for the given set of observations.

In estimating parameters from the observations, there is the possibility of nonuniqueness. This implies that the information matrix is not of full rank or is ill conditioned. To account for this, a singular value decomposition[14] is applied to derive a stable, pseudo-inverse. The optimization algorithm can be summarized as follows:

1) Take an initial value for the parameter vector, say p_0; set counter $k=0$.

2) Evaluate the parameter-step vector Δp, using the modified Gauss-Newton method.

3) Set $p_{k+1} = p_k + \Delta p$.

4) Convergence check; stop if converged; otherwise set $k = k+1$ and return to step 2.

A detailed description of the algorithm is given in Ref. 10.

Definition of Model Parameters

The parameter vector to be estimated is defined as follows:

$$p = [r e Iy_B/m Iz_B/m Iyz_B/m R_l d_x/m d_Y$$

$$\div m d_M/m d_N/m x_1 \psi_M \theta_M \varphi_M \zeta_M v_o]^T$$

The parameters r, e, Iy_B/m, Iz_B/m, and Iyz_B/m represent the inertia characteristics of the system. Accurately estimating them is the main objective of the experiment. The remaining parameters are included in order to explain the measurement data adequately. If one of these parameters is neglected, or inadequately estimated by means of a separate experiment, the estimates of the inertia characteristics will be generally biased.

R_l denotes the effective pendulum length for the lateral motion. As mentioned in the discussion of the suspension rig, the hinges rotate discontinuously for large amplitudes of the lateral motion, which is difficult to account for in a mathematical model. For small amplitudes, the lateral motion is possible only by a bending of the pendulum arms. The resulting Coulomb friction introduces small but not insignificant nonlinearities. Fortunately, as was seen in the equations of motion (3), the lateral motion is represented by a decoupled mode of oscillation (the lateral-sway mode), described by a simple second-order differential equation. By substituting R_l for the effective pendulum length in this equation, and by keeping the excitation of the lateral-sway mode small, it should be possible to estimate this mode separately and thus to reduce the interference of the nonlinearities with the longitudinal sway, pitch, and yaw mode data. For this purpose R_l is included in the above parameter vector. The remaining dimensions of the bifilar pendulum, as defined by the pendulum length R_o and mutual distance $2d$, can accurately be determined by direct measurement and are therefore not included in the parameter vector.

The moments and product of inertia and the damping coefficients appear in the model equations only in proportion to the mass of the aircraft (m). A separate weighing is therefore required. The damping characteristics are included in the model merely to obtain a better data fit, rather than to identify the aerodynamic behavior of the system.

For fixed values of the preceding parameters, the initial state of the system, represented by the eight components of x_1, determines the system response $x(t)$. Computation of the system output, and hence the system response, is required in the optimization algorithm. Since x_1 cannot be assumed known, it has to be estimated simultaneously with the remaining parameters.

For a given system response, the orientation and position of the measurement reference frame F_M, represented by the three Euler angles ψ_M, θ_M, and φ_M and the position vector ζ_M, determine the system output $y(t)$. The orientation and position of F_M can be measured prior to the estimation process. However, these measurements are rather cumbrous and therefore perhaps inaccurate. Including the above parameters in the parameter vector may eliminate the need for the separate measurements and thus may simplify the overall experiment and improve the final results.

The vector v_o represents the bias errors in the observation model (7). Calibration of the instrumentation system may yield the necessary corrections; however, estimation of v_o from the oscillation measurements may yield verification of the calibrations.

Conversion of Parameters to Aircraft Inertia Characteristics

As mentioned, the parameters r, e, Iy_B/m, Iz_B/m, and Iyz_B/m represent the inertia characteristics of the system. The actual coordinates of the center of gravity (cga) with respect to the aircraft datum reference frame F_A (elements of vector ζ_{cga}, see Fig. 4) can be derived from the estimates of r, e, and the nominal aircraft-pitch angle θ_A, where θ_A can be derived from estimates of θ_M (F_A is chosen such that $\theta_A = \theta_M$). Details of the derivations and the necessary corrections for the experimental components (the suspension beam and the suspension frame, see Fig. 1) are given in Ref. 10.

Due to the constraints on roll motion, the mathematical model (and thus the corresponding vector of parameters) does not contain the inertia moment and products related to the roll axis. In order to determine all moments and products of inertia with the present suspension system, different oscillation measurements must be performed with different nominal aircraft pitch angles. The actual moments and products of inertia may then be derived from the different sets of parameter estimates by applying an inertia-axes transformation formula as derived in Ref. 10:

$$J_\theta^T I_A J_\theta = I_B - I_{\exp} \tag{9}$$

where I_B denotes the inertia matrix, which contains all moments and products of inertia with respect to F_B, $I_{\exp}$ denotes the contributions of the experimental components to I_B, I_A denotes the inertia matrix with respect to the body axes (parallel to F_A) through the aircraft center of gravity (cga), and J_θ denotes the transformation matrix for the nominal pitch rotation θ_A. With the estimates of Iy_B, Iz_B, and Iyz_B available from k different oscillation measurements, the following expressions are obtained:

$$\begin{aligned} &Iy_A &&= \hat{I}y_B^i - Iy_{\exp}^i \\ &Iyz_A \cos\hat{\theta}_A^i - Iyx_A \sin\hat{\theta}_A^i &&= \hat{I}yz_B^i - Iyz_{\exp}^i \\ &Iz_A \cos^2\hat{\theta}_A^i + Ixz_A \sin 2\hat{\theta}_A^i + Ix_A \sin^2\hat{\theta}_A^i &&= \hat{I}z_B^i - Iz_{\exp}^i \quad i=1,\ldots,k \end{aligned} \tag{10}$$

where the symbol (^) is used to denote the quantities directly estimated from the oscillation measurements. Note that the contributions of the experimental components on the right-hand sides depend on the orientation of the aircraft in equilibrium position. Details of the derivations are given in Ref. 10. From the last equation of the above set (with unknowns Iz_A, Ixz_A, and Ix_A), it follows that at least three different oscillation measurements with different nominal pitch angles must be carried out to derive all aircraft moments and products of inertia.

Simulation

Identifiability

The objectives of simulation are to obtain insight into the identifiability of the parameter vector and to investigate the influence of initial states on the estimation accuracies. The necessary simulation experiments are described in detail in Ref. 10. Simulated response measurements are obtained by adding computer-generated statistical errors to the model output.

The identifiability study shows that the parameter representing the lateral position of the inertia measurement unit (the y coordinate of ζ_M) is strongly correlated to the parameter e. The y coordinate of ζ_M is therefore excluded from the parameter vector and must be measured separately.

The components of v_o corresponding to the specific-force measurements are strongly correlated to the respective components of y_o [cf. Eqs. (6-8)], representing the (constant) gravity contributions to the specific forces. This correlation results in poor estimates of the Euler angles φ_M and θ_M. To obtain accurate estimates of these angles, the bias errors in the specific-force measurements are omitted. A sensitivity study shows that this results in a bias in the estimates of φ_M and θ_M of maximally 0.005 and 0.01 deg, assuming that the omitted bias errors are of the same order as the measurement resolutions.

The simulation experiments show that the response measurements contain sufficient information to obtain independent estimates of the remaining parameters, provided the initial state is properly chosen.

Excitation

Because both the initial position and the initial velocity components are included in the parameter vector (contained in the initial state vector x_1), excitation is not restricted to static initial conditions. The selection of the initial state may be used as a tool for experiment optimization. The influence of the choice of initial conditions on the estimation accuracies is investigated by means of a closer examination of the information matrix.[13] Because it is difficult to establish precisely a given initial state in the actual experiment, the objective of these investigations is to take an overall view of the effects of different excitations on the estimation accuracies rather than to find the optimal initial state precisely.

It is seen that excitation of both the longitudinal-sway and yaw modes is of first importance in obtaining accurate estimates of all parameters. Excitation of the pitch mode is not of primary importance. Domination of the longitudinal motion is required to obtain sufficient independent information about the vertical position of the center of gravity (the parameter r) and the pitching moment of inertia (Iy). Of course, the presence of the yaw mode is important to obtain accurate estimates of the yawing moment of inertia (Iz). Identification of the inertia coupling of the system, represented by the parameters e and Iyz, requires that both the symmetrical and asymmetrical motions are present in the response measurements. Excitation of the lateral motion can be kept small, because the corresponding mode is completely decoupled and because observations of this mode yield no additional information about the aircraft inertia characteristics (R_l and d_Y/m are the only parameters in the corresponding equation of motion).

In conclusion, accurate estimates of all parameters can be expected from response measurements in which all modes, except the lateral-sway mode, are about equally present. Excitation of the lateral-sway mode can be kept small, which is desirable to reduce the nonlinear effects caused by the bending of the pendulum arms.

Experimental Results

In order to gain insight into the accuracy that can be expected in practice, the current method is applied to the DHC-2 Beaver, the laboratory aircraft of the Aerospace Department in Delft. It turns out to be relatively simple to swing this aircraft by hand. To obtain the required presence of the modes in the oscillations, the aircraft is persistently excited until the desired combination of modes is obtained. Suppression of the lateral-sway mode requires extra attention.

Several oscillation measurements are conducted for each of the three different nominal aircraft pitch angles in order to determine the reproducibility of the method. The experiments are repeated, first with a weight of known mass placed on the left wing (load condition B) and then with the weight on the rear wheel (load condition C), in order to simulate different loading configurations (the unloaded aircraft is denoted by load condition A).

For the purpose of verification, an additional, more conventional experiment[10] is used to determine the vertical and longitudinal location of the aircraft center of gravity. This experiment is based on the measurement of changes in the nominal pitch angle caused by a weight attached to the tail, with the aircraft suspended in the bifilar pendulum construction.

Before the numerical results are presented and discussed, the mathematical model is verified against the data collected from the system.

Model Verification

The mathematical model is verified by a residual analysis. The residuals are the differences between the actual measurements and the measurements according to the identified (estimated) model. If the residuals are large and do not show the expected "noisy" character (measurement resolution), they indicate errors in the model formulation and/or excessive bias in the parameter estimates.

As a consequence of too large amplitudes of the oscillations applied in the experiments, effects could clearly be observed in the residuals that can be attributed to second-order nonlinearities (products of state components neglected in the linearized mathematical model). Fortunately, these second-order elements can be included in the mathematical model by adding the proper elements to the vector w in the state-space equation (5) and to the output equation (6). It must be noted that these extensions (which required rather time-consuming algebra and some perseverance) can be avoided if the amplitudes of the oscillations are kept smaller.

The measurement signals and the residuals of the extended model for a given measurement registration of 20 s are shown in Fig. 5. The rotation-rate measurements about the X_M axis are mainly explained by the above (small) second-order nonlinear elements. Variations in the specific forces along the Z_M axis are small as compared to the resolution of the corresponding measurements.

The coupled longitudinal-sway and pitch modes are clearly visible in the measurements of the specific forces along the X_M and Z_M axes and rotation rates about the Y_M axis. For the present system, the natural frequency of the longitudinal-sway mode is about twice the natural frequency of the pitch mode. The yaw and lateral-sway modes dominate the rotation rates about the Z_M axis and the specific forces along the Y_M axis, respectively. Apparently, the lateral-sway mode is not sufficiently suppressed.

The specific-force residuals are of the same magnitude as the corresponding measurement resolutions (they show the expected noisy character). The rotation-rate residuals, though good convergence of the estimation process is attained, still tend to show coherent phenomena. The underlying model errors can be attributed to a violation of the assumptions concerning the suspension construction (see Ref. 10 for a more extensive discussion).

First, due to the relatively large amplitudes of the lateral motion, and because the natural frequencies of the lateral-

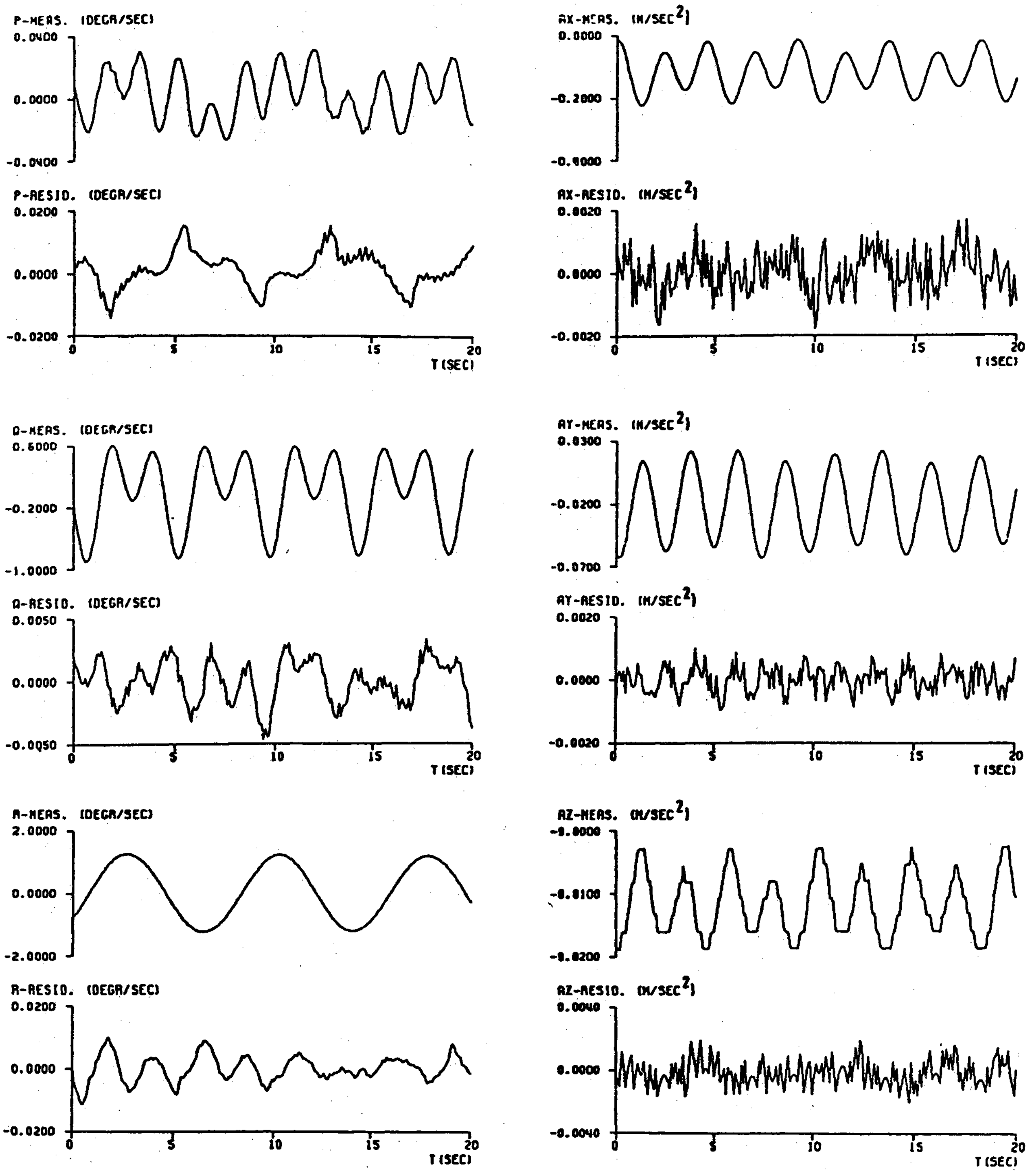

Fig. 5 Measurement signals and residuals for a 20-s observation period.

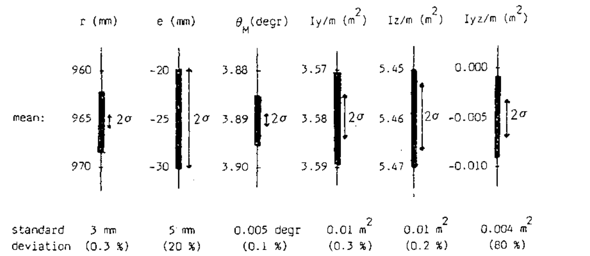

Fig. 6 Graphical representation of the estimation results and corresponding square roots of Cramer Rao lower bounds ($\sigma_s/2$) for 19 oscillation experiments (only the most relevant parameters are indicated).

sway and longitudinal-sway modes do not differ much, the lateral motion cannot be adequately isolated from the measurement data, that is, not with the present mathematical model. A more detailed evaluation of the model is required with respect to the present kinematic constraints. To allow a better data fit, and to simplify the excitation of the system in future experiments, a modification of the hinges for the lateral motion is recommended.

Second, due to the relatively large mass of the pendulum arms (about 15 kg each) compared to the mass of the aircraft (about 2000 kg), the kinetic energy and changes in the potential energy of the pendulums contribute significantly to the overall dynamics of the system. These contributions are not explained with the present mathematical model (where the mass of the pendulums is neglected), which results in small but not insignificant residuals. It must be noted that the above effects can only be observed because of the excellent accuracy of the instrumentation system.

Numerical Results

Despite the relatively large residuals, the parameter estimates are accurately reproduced from the different oscillation measurements. The most relevant results are discussed here. Details can be found in Ref. 10.

Figure 6 is a graphical representation of the estimation results for 19 oscillation experiments. All parameter estimates lie within the areas indicated by the vertical bars. The Cramer Rao lower bounds of the estimator deviations[13] are denoted by σ_s. The difference between these estimator deviations and the actual deviations can be attributed to model errors.

All estimates of r, Iy_B/m, and Iz_B/m agree within 0.3% with the respective mean values. The standard deviations of estimates of e and Iyz_B/m are smaller than 5 mm and 0.004 m^2, respectively. Deviations in estimates of θ_M are smaller than 0.01 deg, which agrees with the conclusions drawn from the simulation experiments.

Constant Earth-rotation components could be identified (see Ref. 10) in the estimates of the rotation-rate bias errors, which justifies the inclusion of the bias errors in the parameter vector (variations in these components due to the aircraft oscillations are smaller than the measurement resolution).

The final results, obtained after the conversion from the parameter estimates to the actual aircraft inertia characteristics, are presented in Table 1. Estimates of the center-of-gravity coordinates are averaged per load condition. The aircraft moments and products of inertia are obtained by evaluating least-squares solutions to the complete set of equations (10) for each load condition separately. To allow a comparison, the results for load conditions B and C are also corrected for the contributions of the additional weight on the left wing and rear wheel, respectively. The results of the more conventional center-of-gravity measurement are given in the table, too.

From Table 1 it is seen that variations in Ix_A are extremely large. This is not a surprising result. The measurement data do not contain enough independent information about the roll characteristics of the aircraft because the different nominal aircraft pitch angles applied in the experiments are too small (currently, about +5, 0, and −5 deg). In mathematical terms, this means that the complete set of transformation equations (10) is ill conditioned with respect to the identifiability of Ix_A. In simpler terms, the coefficients of Ix_A in the transformation equations ($\sin^2\theta_A$) are too small. The same holds true, though to a lesser extent, for the remaining roll characteristics, Ixz_A and Ixy_A.

To obtain better results for the roll characteristics, the three nominal aircraft orientations should be chosen much farther apart. However, the choices of nominal pitch angles are restricted in the present suspension construction. Moreover, the nominal pitch angles needed to obtain sufficient information about the roll characteristics require that the aircraft be hoisted in a very abnormal fashion [at least +45 and −45 deg, as can be seen from a sensitivity analysis of the transformation formulas (10)]. Therefore, it is recommended that the nominal yaw angle be varied instead of the nominal pitch angle. Similar transformation formulas can be derived for the determination of the roll characteristics in that case. Although the mathematical model can be left unchanged, the suspension construction should be slightly modified.

The results of the more conventional method for the determination of the vertical location of the center of gravity, though the standard deviations of this method are estimated to be more than 10 mm (Ref. 10), confirm the significant contributions of the pendulum arms to the overall inertia characteristics, as was indicated in the discussion about the residuals. The mass of the lower links of the pendulums (about 7 kg each) is accounted for in the conversion from parameter estimates to final results; these links are considered fixed to the suspension beam. The remaining bias can be explained only by including the dynamics (and not simply the mass) of the complete pendulums in the mathematical model. Once more, it must be noted that these relatively small effects can be observed only because of the high reproducibility of the method, enforced by the high-accuracy of the instrumentation system.

Table 1 Inertia characteristics for three load conditions and results after corrections for test weights on left wing (B) and aircraft tail (C). Dimensions in m (for cg) and Kg-m^2 (for I_A); aircraft mass: 1916 Kg; and aerodynamic chord: 1.6 m.

Load condition	$\zeta(x)$ cga	$\zeta(y)$ cga	$\zeta(z)$ cga	Ix_A	Iy_A	Iz_A	Iyz_A	Ixy_A	Ixz_A
A	0.532	0.025	−0.781	3827	6957	10782	−6	−38	316
B	0.533	0.102	−0.768	8287	7000	11691	168	−23	445
B-corrected	0.533	0.029	−0.777	7320	6988	10736	60	−24	445
C	0.596	0.025	−0.784	4417	7692	11534	−9	−29	313
C-corrected	0.533	0.025	−0.780	4413	6953	10799	−10	−25	365
A (conv)	0.534	—	−0.801	—	—	—	—	—	—

Discussion

The experimental results presented in the previous section indicate a potential improvement with respect to accuracy as compared to the more conventional methods. The highest accuracies reported so far are of the order of 1% for the inertia moments *Iy* and *Iz* and the vertical location of the center of gravity,[6] whereas these quantities are reproduced with an accuracy of 0.3% with the current method. Corrections are yet to be made for the contributions of additional air mass,[15] which may, at worst, eliminate the accuracy improvement.

But, it is the simplicity of the method rather than the improved accuracy that we want to stress. The conventional methods require time-consuming rig reconfigurations, separate center-of-gravity measurements and, most important, extreme care in conducting the various experiments. With the method presented in this paper, the only separate experiment required is the measurement of the pendulum dimensions (R_0 and d) and the lateral position of the instrumentation system (and the assessment of the inertia characteristics of the experimental components, but this is required in the conventional methods, too). These quantities have to be measured only once; they are independent of the loading configuration of the aircraft (tanks full/empty, pilot, passengers, etc.). Moreover, these measurements are relatively simple and can accurately be carried out. Note that, in the conventional methods, all separate experiments must be repeated for each loading configuration.

The actual experiment is very simple in operation: suspend the aircraft, switch the instrumentation system on, and give the aircraft a push. The more complicated part of the method, i.e., the parameter estimation, is done by the computer.

The following suggestions are indicated in order to further establish the method as a practical tool:

1) Include the dynamics of the pendulum arms in the derivation of the mathematical model. This should eliminate the observed estimation bias for the vertical location of the center of gravity.

2) Modify the hinges for the lateral motion (use knife edges to reduce friction). This eliminates the restrictions on excitation with respect to the lateral motion, simplifying the experiment.

The above two actions will, in addition, reduce the model errors and thus increase the reproducibility of the estimation. Finally:

3) Modify the suspension construction such that the aircraft can be suspended at different nominal yaw angles (e.g., 90, 45, and 0 deg).

The last suggestion offers the possibility of estimating all inertia characteristics from three different oscillation measurements without the need for rig reconfigurations. However, it may be interesting to investigate the possibilities offered by the application of the method to a suspension system in which the aircraft is allowed to oscillate about all three axes simultaneously. The currently obtained results give good reason to believe that, in such a case, all inertia characteristics can simultaneously be estimated from the measurements of only one oscillation. The great advantages of such an application are obvious.

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